Analysis of UPML Design Parameters in the Solution of Electromagnetic Scattering Problems

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Abstract - In this work, the UPML design parameters are analyzed for solving electromagnetic scattering problems using the Finite Element Method (FEM). Analysis are presented for the maximum conductivity and thickness of the absorbent layer. The results show that small values of the conductivity may not adequately represent the decay of the fields while high values may generate reflections. The thickness of the layer, in turn, must have a minimum value to guarantee the convergence of the solution.

***Index Terms*— UPML, FEM, Electromagnetic scattering.**

I. INTRODUCTION

The use of the Finite Element Method (FEM) to solve unlimited problems, such as electromagnetic scattering, has been the reason of several studies and research. Due to the characteristics of this type of problems it is necessary to introduce an artificial boundary to limit its computational domain. The insertion of this boundary can generate wave reflections in the study domain [1]. An efficient approach to dealing with this drawback is to use a perfectly matched absorbent layer (PML) as an artificial border [2].

Initially proposed for the FDTD, the UPML (Uniaxial Perfectly Matched Layers) began to be employed in the FEM for truncation of the computational domain of open problems [1]. The UPML is based on the use of absorbent layers and on the concept of absorbing, theoretically without spurious reflections, the electromagnetic wave at any frequency and incidence angle [3].

This paper aims to present the behaviour of UPML design parameters for electromagnetic scattering problems solved via FEM. The analysis and construction characteristics of each parameter of the project are presented. The results show that for an efficient absorption, adequate parameteres values should be used.

II. UPML

An anisotropic PML medium is known as UPML and has been initially proposed by Sacks [2]. For a single interface, the anisotropic medium is uniaxial and composed of magnetic and electric tensors. This medium works as the PML proposed by Berenger avoiding undesirable nonphysical field [2].

A. Tensors

The construction of the UPML medium is based on the definition of the magnetic and electric tensors, which can be defined as [3]:

$$\bar{\bar{\varepsilon}} = \varepsilon\bar{\bar{s}} \qquad \bar{\bar{\mu}} = \mu\bar{\bar{s}}; \qquad \bar{\bar{s}} = \begin{bmatrix} \frac{s_y s_z}{s_x} & 0 & 0 \\ 0 & \frac{s_x s_z}{s_y} & 0 \\ 0 & 0 & \frac{s_x s_y}{s_z} \end{bmatrix} \qquad (1)$$

where, ε is the electric permissivity of the medium, μ is the permeability of the medium and s_x, s_y and s_z are the components of the tensor $\bar{\bar{s}}$.

Similar to the Berenger's PML, the unreflective property of the UPML medium is valid for any s_x, s_y and s_z.

B. The incorporation of the UPML into the FEM

The incorporation of the UPML into the finite element method can be done by inserting the tensors given by (1) into the Maxwell equations [3]. Thus, the wave equation for the magnetic field can be defined as [1]:

$$\nabla \cdot (\alpha_1 \nabla \boldsymbol{H_z}) + k_0^2 \alpha_2 \boldsymbol{H_z} = 0 \qquad (2)$$

where, $\alpha_1 = \mu_r^{-1}\bar{\bar{s}}^{-1}$, $\alpha_2 = \varepsilon_r\bar{\bar{s}}$ and k_0 is the wave number.

Based on (2), the weak FEM-UPML formulation for an electromagnetic scattering problem due to a conductive object can be derived and given by Eq. (3) [1].

$$\int_\Omega \nabla w.(\alpha_1\bar{\bar{s}}\nabla u^s) - k_0^2\alpha_2\bar{\bar{s}}w.u^s d\Omega = -\int_\Upsilon w\frac{\partial u^i}{\partial n}\, d\Upsilon \qquad (3)$$

where, w is the weighting function, u^s is the scattered magnetic field, Υ is the object surface and u^i is the incident magnetic field.

To ensure that the smallest reflection at the termination occurs, a perfect conductor (PEC) is used as termination of the absorber layer [4].

C. UPML Project

The construction of the UPML is done by means of the definition of some parameters present in the tensors, among them stand out: i) the degree of gradation (m) that represents the way that the conductivity (σ) varies within the layer; ii) the maximum conductivity (σ_{max}). It is necessary to find a balance of this parameter since for a very small value of σ_{max} the attenuation of the field is not enough to eliminate the reflection. On the other hand, high values of σ_{max}, reflections can occur, since the finite element mesh is insufficient to model the quickly change in material properties; iii) the scaling factor k is present in the equation for calculating the tensors s_x and s_y . The increase of k reduces the phase velocity of the wave which leads to an artificial increase of the refractive index and improves the performance of the UPML in cases of oblique incidence; iv) the reflection factor (R); and, v) the thickness of the layer.

The arguments of the tensor can be given by [1]:

$$s_x = k_x - \frac{j\sigma_x}{\omega\varepsilon_1}, \; s_y = k_y - \frac{j\sigma_y}{\omega\varepsilon_1}. \qquad (4)$$

The tensor arguments $\bar{\bar{s}}$ have distinct values in different areas of the mesh, as depicted by Fig. 1. In the area of intersection between the planes x and y $s_z = 1$, between x_{min} and x_{max} $s_y = s_z = 1$ and between y_{min} and y_{max} $s_x = s_z = 1$. Note that for a two-dimensional problem s_z is always equals 1. Within the mesh, that is, outside the absorbers $s_x = s_y = s_z = 1$.

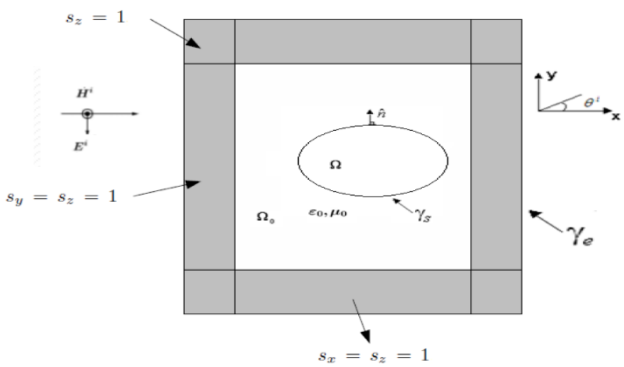

Figure 1: UPML representation and tensors value in each region.

Several profiles are suggested for σ and k gradations. The most efficient ones are those that use the polynomial variation in the UPML environment [4]:

$$\sigma_x = \left(\frac{x}{L}\right)^m \sigma_{max} \qquad \sigma_y = \left(\frac{y}{L}\right)^m \sigma_{max} \tag{5}$$

$$k_x = 1 + (k_{max} - 1)\left(\frac{x}{L}\right)^m \qquad k_y = 1 + (k_{max} - 1)\left(\frac{y}{L}\right)^m \tag{6}$$

where x and y are the positions of the element in the layer, L is the inner side of the layer, m is the degree of gradation of the polynomial and σ_{max} is the maximum value of conductivity that can be given by:

$$\sigma_{max} = -\frac{c\varepsilon_0 \ln(R)}{\left(\frac{2}{m+1}\right)L}. \tag{7}$$

In (7) ε_0 represents the electric permissivity in the vacuum, c is the speed of light, R is the desired reflection, n the degree of gradation and L the thickness of the layer.

III. RESULTS

The problem of electromagnetic scattering due to a 2D perfect conductor is solved via FEM-UPML. For all analysis of UPML parameters, the following datas are considered: cylinder radius of 0.3λ, distance between the target and the absorber of 0.1λ and layer thickness of 1.6λ.

Table 1 shows the average error for the absolute value of the magnetic field obtained from the FEM-UPML for various values of m and the analytical solution [1]. It is observed that $m = 3$ gives the smallest error.

Table 1

Absolute average error of the magnetic field for various values of m.

m	1	2	3	4	5
Absolute average error	3.91%	0.94%	0.67%	1.07%	1.69%

Table 2 shows the average error for the absolute value of the magnetic field obtained from the FEM-UPML for various values of k_{max} and the analytical solution. The best result is obatained for $k_{max} = 6$.

Table II

Relative average errors for variation of parameter k_{max}.

k_{max}	1	3	6	10
Relative average error	6.6%	4.12%	3.7%	5.04%

Figure 2 shows the behavior of the magnetic field within the layer for various values of σ_{max}. It is observed that for σ_{max} between 0.5 and 1, the field is totally absorbed by the absorbent layer. And, for very small values of σ_{max} the fields presents some oscillation and are not well absorbed.

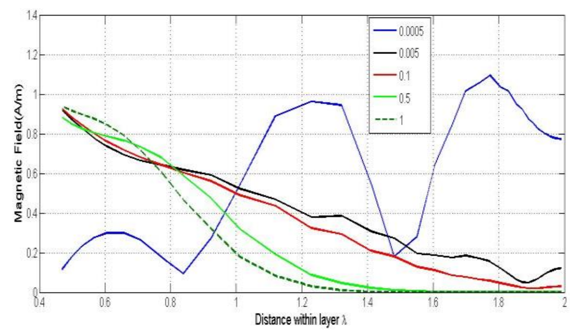

Figure 2: Field behavior analysis due to variation σ_{max}.

To evaluate the influence of the thickness of the UPML layer, it is varied from 0.4λ to 3.6λ with step of 0.1. Figure 3 presents the results. It can be noticed that for the studied case, the thickness of 1.2λ reaches the minimum error.

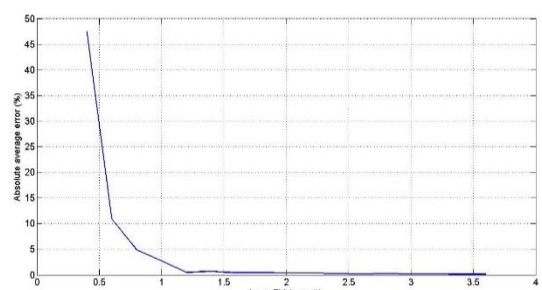

figure 3: Absolute Average Error x Layer Thickness.

IV. CONCLUSION

This paper presents a study of the variation of UPML design parameters for electromagnetic scattering problems using FEM. From the results, it is possible to conclude that the UPML is very sensitivity of its parameters values.

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